A Text Book on

Electro-Magnetism

Magnetic Field Generated by Rotating Electric Charge

Author

Harish Chandra Rajpoot

Doctor of Philosophy

Indian Institute of Technology Bombay

This book is dedicated to my loving parents, elder brother Dr. Chandra Shekhar Rajpoot & Mr Upendra Sir for their love, affection, inspiration & invaluable sacrifice &

To The Learners

With hope that this book will develop interest & skill in

Electro-Magnetism

Introducing a mathematical approach

Preface

The concept of electro-magnetism still needs to be mathematically more improved & simplified to make its feasible applications in the field of Physics. Mathematical derivations & formulations of Electro-magnetism become essential for its applications in basic & applied Physics & so in the field of Engineering. Especially Geometry, calculus, trigonometry have been applied to analyse various articles in Electro-magnetism. The learners studying at any level or higher education graduate & undergraduate can find it useful to learn the basic & advanced mathematical concepts in Electro-magnetism. This academic book especially deals with the Magnetic Field & Magnetic Moment produced by rotating electric charge in Electro-magnetism & hence is available as a reference book.

The author uses the analogy of a uniformly rotating electric charge to the electric current. A point electric charge moving with a constant speed along a circular path of certain radius behaves like a circular loop of electric current with the same radius, its centre at the point of rotation and its plane coincident with the plane of rotation of point charge. If an electric charge is uniformly distributed over a line, surface or throughout a solid rotating at a constant angular speed about an axis then every point (i.e. differential charge) on that body also rotates in the same direction, at the same angular speed about the same axis. The differential amount of electric charge concentrated at a point and rotating at a constant angular speed behaves like a circular loop of tiny current generating differential magnetic field along the axis of rotation. This concept and analogy is used to derive an analytic master formula (also in differential form), using Biot-Savart Law, for further deriving the numerous mathematical expressions to analytically compute the strength of magnetic field, at the axis of rotation, generated by a point or uniformly distributed electric charge over a straight line, a ring, a disk (lamina), a curved surface or throughout a solid body (like cube, cylinder, sphere etc.) rotating with a constant angular speed about an axis. The magnetic field generated by a uniformly rotating electric charge is along axis of rotation, direction of which is given by Right Hand Rule.

Easy mathematical derivations, numerical examples & key-points are salient features of the entire book. The author thinks that the present edition will be very useful to the learners to develop new skills in this subject. Although the best efforts have been made by the author to make this book error free still some errors might have crept in. Therefore, all the learners & readers are welcome to give their feedback to the author.

H. C. Rajpoot

Acknowledgements

After a long & deep study, I could complete this book (created as an academic research) written on my interesting subject “Electro-magnetism”. I had to study & work constantly for a long time on this book for new findings in this subject. I offer my great sense of gratitude to my parents & elder brother Dr. C. S. Rajpoot who always, loved, inspired, encouraged, helped & have been showering their blessings on me. Also, I am thankful to all my teachers specially Mr Upendra Sir, an experienced teacher of physics at Oxford Model I.C. Syam Nagar, Kanpur, who not only taught & guided but also helped me with all the aspects in my studies & always inspired me to bring innovative ideas in the field of scientific and technical education. He had been my best guide & directed me for this work. The author is deeply thankful to ALMIGHTY GOD who kept him physically fit during the completion of this academic research based book.

Hoping my work will be useful for all the learners & wishing them a profitable & enjoyable journey through this book.

All the derivations & numerical data of this book computed by the author Mr H.C. Rajpoot, are mathematically correct to the best of his knowledge & experience.

Interior Files

1. A positive electric charge Q is uniformly distributed over a straight line-segment AB (i.e. coincident with X-axis) such that the distances of its end-points A and B from the origin O are a & b ($\forall\, a < b$) respectively (as shown in fig-4). If it is rotated at a constant angular speed ω about Y-axis (i.e. lying in XY-plane of paper), magnetic field B generated at the origin O (i.e. centre of rotation) is given by following formula (1)

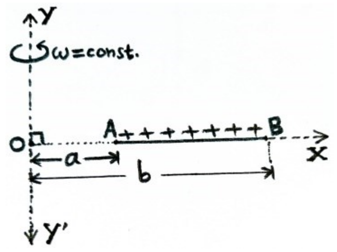

Figure 4: A line-segment AB rotating with a constant angular speed ω about Y-axis in XY plane (i.e. the plane of paper)

$$\mathbf{B = \frac{\mu_o}{4\pi}\frac{\omega Q}{(b-a)}\ln\left(\frac{b}{a}\right)} \quad \dots\dots\dots(1)$$

The ratio of magnetic dipole moment (M) & angular momentum (J) of an electric charge uniformly distributed over a rotating line-segment: Consider an electric charge Q of mass m uniformly distributed over a line segment AB of length $(b-a)$ rotating at a constant angular speed ω about the centre O (as shown in fig-5 above). From above equations (1.1) & (1.2), the ratio of magnetic dipole moment M and angular momentum J is given as follows

$$\frac{M}{J} = \frac{\left(\frac{\omega Q(a^2+b^2+ab)}{6}\right)}{\left(\frac{m(a^2+b^2+ab)\omega}{3}\right)} = \frac{Q}{2m}$$

$$\mathbf{\frac{M}{J} = \frac{Q}{2m}}$$

Above is the standard formula for the ratio of magnetic dipole moment M and angular momentum J of a uniformly distributed & rotating electric charge Q of mass .

Solved examples based on Mathematical Formula in Electromagnetism

The directions of magnetic field ($\vec{B}$) & magnetic dipole moment ($\vec{M}$) are same along the angular velocity vector ($\vec{\omega}$) for a positive electric charge but both are opposite to $\vec{\omega}$ for a negative electric charge. The direction of angular momentum ($\vec{J}$) is always along the angular velocity vector ($\vec{\omega}$) irrespective of the nature of electric charge. The specific formula (as derived above) will be recalled and applied to solve a particular numerical problem. Therefore it's advisable to go through all above formula and derivations for solving the numerical questions related to Electromagnetism as given below.

Q1. Two electric charges +2C & -7C are uniformly distributed over the lengths AC=4cm and BC=8cm respectively of the line AB which rotates at a constant angular speed $5\pi\, rad/sec$ such that the normal distance of its end-point A from the axis of rotation YY' is 2cm (As shown in fig-48). Find out the strength of magnetic field (B) at the axis of rotation and the magnetic dipole moment (M).

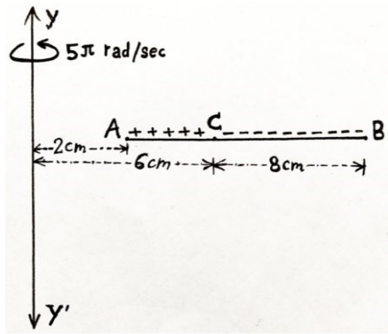

Figure 48: Two charges +2C and -7C are distributed uniformly over the lines AC & BC respectively of line AB rotating ⊥ to the plane of paper

Sol. There are two electric charges of opposite nature distributed over different lengths of line AB therefore we will compute the magnetic field & magnetic dipole moment generated by individual electric charge as follows

The magnetic field $|\vec{B_1}|$ generated by electric charge $Q = +2\ C$ uniformly distributed over the line-segment AC rotating at a constant angular speed $\omega = 5\pi\ rad/sec$ about axis YY' such that the normal distances of end-points A & C from the axis of rotation YY' are $a = 2\ cm\ \&\ b = 6\ cm$ respectively (see fig-48) is given from formula(1) (as derived above) as follows

$$\boldsymbol{B = \frac{\mu_o}{4\pi}\frac{\omega Q}{(b-a)} ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow\ |\vec{B_1}| = 10^{-7}\cdot\frac{5\pi\cdot 2}{(6-2)\times 10^{-2}} ln\left(\frac{6\times 10^{-2}}{2\times 10^{-2}}\right) \qquad (Substituting\ corresponding\ values)$$

$$|\vec{B_1}| = 2.5\pi\times 10^{-5}\ ln\,3 \approx 8.628\times 10^{-5}\ T\ \ (collinear\ with\ \vec{\omega}\,)$$

& magnetic dipole moment $|\vec{M_1}|$ of uniformly charged line AC is given by substituting $\omega = 5\pi\ rad/sec$, $Q = 2\ C,\ a = 2\ cm = 2\times 10^{-2}\ m$ & $b = 6\ cm = 6\times 10^{-2}\ m$ in above equation(1.1) as follows

$$\boldsymbol{M\ = \frac{\omega Q(a^2+b^2+ab)}{6}}$$

$$\Rightarrow |\vec{M_1}|\ = \frac{5\pi\cdot 2\big((2\times 10^{-2})^2 + (6\times 10^{-2})^2 + (2\times 10^{-2})(6\times 10^{-2})\big)}{6}$$

$$\Rightarrow |\vec{M_1}|\ = \frac{260\pi\times 10^{-4}}{3} \approx 0.027227\ Am^2\ \ \ (collinear\ with\ \vec{\omega}\,)$$

The magnetic field $|\overrightarrow{B_2}|$ generated by electric charge $Q = -7C$ uniformly distributed over the line-segment BC rotating at a constant angular speed $\omega = 5\pi\ rad/sec$ about axis YY' such that the normal distances of end-points C & B from the axis of rotation YY' are $a = 6\ cm$ & $b = 14\ cm$ respectively (see above fig-48) is given from formula(1) (as derived above) as follows

$$\boldsymbol{B = \frac{\mu_o}{4\pi}\frac{\omega Q}{(b-a)} ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow\ |\overrightarrow{B_2}| = 10^{-7}\cdot\frac{5\pi\cdot 7}{(14-6)\times 10^{-2}} ln\left(\frac{14\times 10^{-2}}{6\times 10^{-2}}\right) \qquad (Substituting\ corresponding\ values)$$

$$|\overrightarrow{B_2}| = \frac{35\pi\times 10^{-5}\ ln\left(\frac{7}{3}\right)}{8} \approx 11.646\times 10^{-5}\ T\ \ (opposite\ to\ \vec{\omega}\,)$$

& magnetic dipole moment $|\overrightarrow{M_2}|$ of uniformly charged line BC is given by substituting $\omega = 5\pi\ rad/sec$, $Q = 7\ C$, $a = 6\ cm = 6\times 10^{-2}\ m$ & $b = 14\ cm = 14\times 10^{-2}\ m$ in above equation(1.1) as follows

$$\boldsymbol{M = \frac{\omega Q(a^2+b^2+ab)}{6}}$$

$$\Rightarrow\ |\overrightarrow{M_2}| = \frac{5\pi\cdot 7\big((6\times 10^{-2})^2+(14\times 10^{-2})^2+(6\times 10^{-2})(14\times 10^{-2})\big)}{6}$$

$$|\overrightarrow{M_2}| = \frac{553\pi\times 10^{-3}}{3} \approx 0.5791\ Am^2\ \ \ (opposite\ to\ \vec{\omega}\,)$$

Now, taking the algebraic sum, the magnetic field $|\vec{B}|$ generated by a uniformly charged line AB at the axis of rotation YY' as follows

$$|\vec{B}| = |\overrightarrow{B_1}| - |\overrightarrow{B_2}|$$

$$= 8.628\times 10^{-5}\ - 11.646\times 10^{-5}$$

$$= -3.018\times 10^{-5}\ \mathrm{T}$$

$$\therefore\ |\boldsymbol{\vec{B}}| = \boldsymbol{3.018\times 10^{-5}\ T\ (opposite\ to\ \vec{\omega}\,)} \qquad \boldsymbol{Ans.}$$

Similarly, taking the algebraic sum, the magnetic dipole moment $|\vec{M}|$ of a uniformly charged line AB rotating at a constant angular speed as follows

$$|\vec{M}| = |\overrightarrow{M_1}| - |\overrightarrow{M_2}|$$

$$= 0.027227 - 0.5791$$

$$= -0.551873\ Am^2$$

$$\therefore\ |\boldsymbol{\vec{M}}| = \boldsymbol{0.551873\ Am^2\ (opposite\ to\ \vec{\omega}\,)} \qquad \boldsymbol{Ans.}$$

Q2. An electric charge +15C is uniformly distributed over a rectangular lamina ABCD with length & width $10\ cm$ & $6\ cm$ which rotates at a constant angular speed $\frac{25}{\pi}\ rad/sec$ about its vertex A (As shown in fig-49). Find out the strength of magnetic field (B) at the point of rotation and magnetic dipole moment (M).

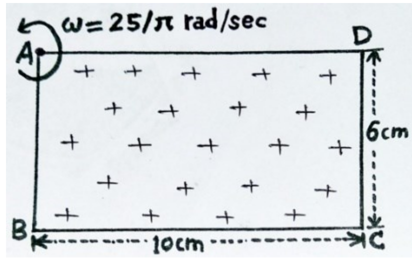

Figure 49: Electric charge +15C is distributed uniformly over the rectangular lamina ABCD rotating at $\frac{25}{\pi}$ rad/sec in the plane of paper

Sol. The magnetic field $|\vec{B}|$ generated by electric charge $Q = +15\ C$ uniformly distributed over the rectangle ABCD of length $l = 10\ cm$ & width $b = 6\ cm$ rotating at a constant angular speed $\omega = \frac{25}{\pi}\ rad/sec$ about its vertex A (see fig-49) is given by formula(25) (as derived above for a rectangular lamina) as follows

$$B = \frac{\mu_o}{4\pi}\frac{\omega Q}{lb}\left(l\ sinh^{-1}\left(\frac{b}{l}\right) + b\ sinh^{-1}\left(\frac{l}{b}\right)\right)$$

$$\Rightarrow |\vec{B}| = 10^{-7}\cdot\frac{\left(\frac{25}{\pi}\right)15}{0.1\cdot 0.06}\left(0.1\ sinh^{-1}\left(\frac{0.06}{0.1}\right) + 0.06\ sinh^{-1}\left(\frac{0.1}{0.06}\right)\right) \quad (Substituting\ corresponding\ values)$$

$$|\vec{\boldsymbol{B}}| = 2.664\times 10^{-4}\ T \ \ (collinear\ with\ \vec{\omega}) \qquad \boldsymbol{Ans.}$$

& magnetic dipole moment $|\vec{M}|$ of uniformly charged rectangular lamina is given by substituting $\omega = \frac{25}{\pi}\ rad/sec$, $Q = 15\ C$, $l = 10\ cm = 0.1\ m$ & $b = 6\ cm = 0.06\ m$ in above equation(25.1) as follows

$$\boldsymbol{M} = \frac{\boldsymbol{\omega Q(l^2 + b^2)}}{\boldsymbol{6}}$$

$$\Rightarrow |\vec{M}| = \frac{\frac{25}{\pi}\cdot 15((0.1)^2 + (0.06)^2)}{6}$$

$$|\vec{M}| = 0.27056\ Am^2 \ \ (collinear\ with\ \vec{\omega}) \qquad \boldsymbol{Ans.}$$

Q3. An electric charge +20C is uniformly distributed over a circular lamina with radius $12\ cm$ which rotates at a constant angular speed $7\ rad/sec$ about its centre. Find out the strength of magnetic field (B) at the point of rotation and magnetic dipole moment (M).

Sol. The magnetic field $|\vec{B}|$ generated by electric charge $Q = +20\ C$ uniformly distributed over the circular lamina of radius $R = 10\ cm = 0.1\ m$ rotating at a constant angular speed $\omega = 7\ rad/sec$ about its centre is given by formula (15) (as derived above for a circular lamina) as follows

$$B = \frac{\mu_o}{2\pi}\frac{\omega Q}{R}$$

$$|\vec{B}| = 2 \times 10^{-7} \cdot \frac{7 \cdot 20}{0.1} \qquad (Substituting\ corresponding\ values)$$

$$|\vec{\boldsymbol{B}}| = 2.8 \times 10^{-4}\ T\ \ (collinear\ with\ \vec{\omega}\ ) \qquad \boldsymbol{Ans.}$$

& magnetic dipole moment $|\vec{M}|$ of uniformly charged circular lamina is given by substituting $\omega = 7\ rad/sec$, $Q = 20\ C,\ \ R = 10\ cm = 0.1\ m$ in above equation(15.1) as follows

$$\boldsymbol{M} = \frac{\boldsymbol{\omega Q R^2}}{\boldsymbol{4}}$$

$$\Rightarrow |\vec{M}| = \frac{7 \cdot 20 \cdot (0.1)^2}{4}$$

$$|\vec{M}| = 0.35\ Am^2\ \ (collinear\ with\ \vec{\omega}\ ) \qquad \boldsymbol{Ans.}$$

Q4. An electric charge -80 C is uniformly distributed over a spherical surface with radius $20\ cm$ which rotates at a constant angular speed $24/\pi\ rad/sec$ about its diametral axis (i.e. passing through the centre). Find out the strength of magnetic field (B) at the axis of rotation and magnetic dipole moment (M) of the spherical surface.

Sol. The magnetic field $|\vec{B}|$ generated by electric charge $Q = -50\ C$ uniformly distributed over the spherical surface of radius $R = 20\ cm = 0.2\ m$ rotating at a constant angular speed $\omega = 24/\pi\ rad/sec$ about its diametral axis, is given by formula (19) (as derived above for a spherical surface) as follows

$$B = \frac{\mu_o}{8}\frac{\omega Q}{R}$$

$$|\vec{B}| = \frac{4\pi \times 10^{-7}}{8} \cdot \frac{\left(\frac{24}{\pi}\right)(-50)}{0.2} \qquad (Substituting\ corresponding\ values)$$

$$= -3 \times 10^{-4}$$

$$|\vec{\boldsymbol{B}}| = 300\ \mu T\ \ (opposite\ to\ \vec{\omega}\ ) \qquad \boldsymbol{Ans.}$$

& magnetic dipole moment $|\vec{M}|$ of uniformly charged spherical surface is given by substituting $\omega = 24/\pi\ rad/sec$, $Q = -50\ C$, $R = 20\ cm = 0.2\ m$ in above equation(19.1) as follows

$$\boldsymbol{M} = \frac{\boldsymbol{\omega Q R^2}}{\boldsymbol{3}}$$

$$\Rightarrow |\vec{M}| = \frac{\left(\frac{24}{\pi}\right) \cdot (-50) \cdot (0.2)^2}{3}$$

$$= -\frac{16}{\pi}$$

$$|\vec{M}| = 5.09296\ Am^2 \quad (opposite\ to\ \vec{\omega}) \qquad \boldsymbol{Ans.}$$

Q5. An electric charge +96 C is uniformly distributed throughout a solid sphere with radius $\mathbf{180\ mm}$ which rotates at a constant angular speed $\boldsymbol{32/\pi\ rad/sec}$ about its axis (i.e. passing through the centre). Find out the strength of magnetic field (B) at the axis of rotation and magnetic dipole moment (M) of the sphere.

Sol. The magnetic field $|\vec{B}|$ generated by electric charge $Q = +96\ C$ uniformly distributed throughout the solid sphere of radius $R = 180\ mm = 0.18\ m$ rotating at a constant angular speed $\omega = 32/\pi\ rad/sec$ about its diametral axis, is given by formula (21) (as derived above for a solid sphere) as follows

$$B = \frac{3\mu_o}{16}\frac{\omega Q}{R}$$

$$|\vec{B}| = \frac{3 \cdot 4\pi \times 10^{-7}}{16} \cdot \frac{\left(\frac{32}{\pi}\right)(96)}{0.18} \qquad (Substituting\ corresponding\ values)$$

$$= 128 \times 10^{-5}$$

$$|\vec{\boldsymbol{B}}| = 1.28\ mT \quad (collinear\ with\ \vec{\omega}) \qquad \boldsymbol{Ans.}$$

& magnetic dipole moment $|\vec{\mathrm{M}}|$ of uniformly charged solid sphere is given by substituting $\omega = 32/\pi\ \mathrm{rad/sec}$, $\mathrm{Q} = 96\ \mathrm{C}$, $\mathrm{R} = 180\ \mathrm{mm} = 0.18\ \mathrm{m}$ in above equation(21.1) as follows

$$\mathbf{M} = \frac{\mathbf{\omega Q R^2}}{\mathbf{5}}$$

$$\Rightarrow |\vec{\mathrm{M}}| = \frac{\left(\frac{32}{\pi}\right) \cdot (96) \cdot (0.18)^2}{5}$$

$$|\vec{\mathrm{M}}| = 6.33645\ \mathrm{Am^2} \quad (\text{collinear with } \vec{\omega}) \qquad \mathbf{Ans.}$$

Q6. An electric charge +100 C is uniformly distributed over the surface of a cube with each side 30 cm which rotates at a constant angular speed 12 rad/sec about the axis passing through the centres of its opposite square-faces. Find out the strength of magnetic field (B) at the axis of rotation and magnetic dipole moment (M) of the rotating cube.

Sol. The magnetic field $|\vec{B}|$ generated by electric charge $Q = +100\text{ C}$ uniformly distributed over the surface of a cube of edge length $a = 30\text{ cm} = 0.3\text{ m}$ rotating at a constant angular speed $\omega = 12\text{ rad/sec}$ about the axis passing through the centres of its opposite faces, is given by formula (26) (as derived above for a cubical surface) as follows

$$B = \frac{2\mu_o}{3\pi}\frac{\omega Q}{a}\sinh^{-1}(1)$$

$$|\vec{B}| = \frac{2\cdot 4\pi\times 10^{-7}}{3\pi}\cdot\frac{(12)(100)}{0.3}\sinh^{-1}(1) \qquad \text{(Substituting corresponding values)}$$

$$|\vec{\mathbf{B}}| = 0.94013\text{ mT} \ \ (\text{collinear with } \vec{\omega}) \qquad \textbf{Ans.}$$

& magnetic dipole moment $|\vec{M}|$ of uniformly charged cubical surface is given by substituting $\omega = 12\text{ rad/sec}$, $Q = 100\text{ C},\ a = 30\text{ cm} = 0.3\text{ m}$ in above equation(26.1) as follows

$$\mathbf{M} = \frac{\mathbf{5}}{\mathbf{36}}\,\boldsymbol{\omega}\mathbf{Q}\mathbf{a}^{\mathbf{2}}$$

$$\Rightarrow |\vec{M}| = \frac{5}{36}(12)(100)(0.3)^2$$

$$|\vec{M}| = 15\text{ Am}^2 \ \ (\text{collinear with } \vec{\omega}) \qquad \textbf{Ans.}$$

Competitive Questions

Subjective Questions

Q1. A point electric charge of magnitude 4π is moving along a unit circle at an angular speed $(12t^2 - 5t + 2)$ at any time t. Find out the strength of magnetic field (B) at the centre of rotation and magnetic dipole moment (M) at time $t = 2$.

Q2. A point electric charge of magnitude $-2C$ is moving along a circle $x^2 + y^2 - 4x + 3 = 0$ in XY-plane, with an angular velocity $(-6t + 5)\hat{k}$ rad/sec at time t sec. Find out the direction and strength of magnetic field (B) and dipole moment (M) at the point (2,0) at time $t = 8$ sec.

(Take dimensions in m & magnetic permeability of vacuum, $\mu_o = 4\pi \times 10^{-7} N/A^2$)

Q3. A point electric charge of magnitude 3π is moving along a circle of radius 3 units. It has an angular acceleration $(12t^2 + 8t)$ at any time t. Find out the strength of magnetic field (B) at the centre of rotation and magnetic dipole moment (M) at any time t. Given that the magnetic field B and magnetic dipole moment are zero at time $t = 0$.

Q4. An electric charge $+4C$ is uniformly distributed over a rectangle defined by $x = \pm 4\ m$ & $y = \pm 2m$ in XY-plane. Now it is rotated about +z-axis CCW at a uniformly increasing angular speed from $\omega = 3\ \mathrm{rad/sec}$ at $t = 0\ sec$ to $\omega = 9\ \mathrm{rad/sec}$ at time $t = 3\ sec$. Find out the magnetic field (B) at the origin (0,0) and dipole moment (M) at time $t = 2\ sec$.

Q5. An electric charge $+12C$ is uniformly distributed over an arc defined by $x^2 + y^2 = 9m^2\ \forall\ x, y \geq 0$ in XY-plane. Now it is rotated CCW from rest (i.e. $\omega = 0$ at $t = 0$) about +z-axis at a uniformly increasing angular acceleration $\alpha = 8t - 3\ \mathrm{rad}/sec^2$ with time t. Find out magnetic field vector ($\vec{B}$) at the origin (0,0) as a function of time t.

Q6. An electric charge $-6C$ is uniformly distributed over a plane bounded by $= 5x$, $x = 2$, & $y = 0$ in XY-plane. This plane is rotated at an angular velocity $\omega = (2t + 10)\hat{k}\ \mathrm{rad/sec}$ at time t. Find out magnetic field vector ($\vec{B}$) at the origin (0,0) at time $t = 100\ sec$. (Take MKS)

Q7. An electric charge $+20C$ is uniformly distributed over all 15 equal sides, each of 20 cm, of a regular polygon with centre at the origin & lying in XY-plane . This polygon is rotated about its centre at an angular velocity $\omega = (t^2 + 2)\hat{k}\ \mathrm{rad/sec}$ at time t. Find out magnetic field ($\vec{B}$) & magnetic dipole moment ($\vec{M}$) at the centre of polygon at time $t = 30\ sec$.

Q8. An electric charge $+5C$ is uniformly distributed over a plane bounded by $x^2 + y^2 - 4x + 6y - 12 = 0$ & $x^2 + y^2 - 4x + 6y - 36 = 0$ in XY-plane. This plane is rotated about a point $(2, -3)$ at an angular velocity $\omega = (2t^2 + 3)\hat{k}\ \mathrm{rad/sec}$ at time t. Find out the rate of change of magnetic field $\left(\frac{d\vec{B}}{dt}\right)$ & dipole moment $\left(\frac{d\vec{M}}{dt}\right)$ at point (2,-3) at time $t = 15\ sec$. (Take MKS)

Multiple Choice Questions

Q1. An equal amount of electric charge is uniformly distributed over a circle and a circular arc each of equal radius. Now, both the circle and circular arcs are rotated uniformly about their centres at an equal angular speed in their planes. If $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$ are the magnetic field vectors at the centres of circle and circular arc respectively then

$a)\ |\overrightarrow{B_1}| > |\overrightarrow{B_2}|$ $b)\ |\overrightarrow{B_1}| = |\overrightarrow{B_2}|$ $c)\ |\overrightarrow{B_1}| < |\overrightarrow{B_2}|$ $d)\ none$

Q2. An electric charge Q units is uniformly distributed over a line segment joining the points $(2, 4)$ & $(6, 12)$. This segment is rotated uniformly about the origin $(0, 0)$ in XY-plane at an angular speed ω such that its orientation does not change. The magnitudes of magnetic field & dipole moment at origin are respectively

$a)\ \frac{\mu_o \omega Q \ln 3}{16\pi\sqrt{5}}, \frac{130\omega Q}{7}$ $b)\ \frac{\mu_o \omega Q \ln 3}{18\pi\sqrt{5}}, \frac{130\omega Q}{7}$ $c)\ \frac{\mu_o \omega Q \ln 3}{16\pi\sqrt{5}}, \frac{130\omega Q}{3}$ $d)\ \frac{\mu_o \omega Q \ln 3}{18\pi\sqrt{5}}, \frac{130\omega Q}{3}$

Q3. A unit electric charge is uniformly distributed over all the sides, each of 2 units, of a regular octagon which is rotating at a constant angular speed ω about its centre in its plane. The magnitudes of magnetic field & dipole moment at the centre are respectively

$a)\ \frac{\mu_o \omega \sinh^{-1}(\sqrt{2}-1)}{4\pi}, \frac{(5+3\sqrt{2})\omega}{3}$ $b)\ \frac{\mu_o \omega \sinh^{-1}(\sqrt{2}+1)}{4\pi}, \frac{(5+3\sqrt{2})\omega}{3}$ $c)\ \frac{\mu_o \omega \sinh^{-1}(\sqrt{2}+1)}{4\pi}, \frac{(5-3\sqrt{2})\omega}{4}$
$d)\ \frac{\mu_o \omega \sinh^{-1}(\sqrt{2}-1)}{4\pi}, \frac{(5-3\sqrt{2})\omega}{4}$

Q4. An electric charge of 2 units is uniformly distributed over a rectangle of length 15 units and width 5 units. The rectangle is rotated at a constant angular speed of 5 units about its centre in its plane. The magnitudes of magnetic field & dipole moment at the centre are respectively

$a)\ \frac{\mu_o}{4\pi}\left\{\sinh^{-1}(3) - \sinh^{-1}\left(\frac{1}{3}\right)\right\}, \frac{260}{3}$ $b)\ \frac{\mu_o}{4\pi}\left\{\sinh^{-1}(3) + \sinh^{-1}\left(\frac{1}{3}\right)\right\}, \frac{260}{3}$

$c)\ \frac{\mu_o}{4\pi}\left\{\sinh^{-1}(3) - \sinh^{-1}\left(\frac{1}{3}\right)\right\}, \frac{250}{3}$ $d)\ \frac{\mu_o}{4\pi}\left\{\sinh^{-1}(3) + \sinh^{-1}\left(\frac{1}{3}\right)\right\}, \frac{250}{3}$

Q5. A unit electric charge is uniformly distributed over a unit circle which is rotating with a time dependent angular acceleration of $at + b$ ($\forall\ a, b > 0$) about its centre in its plane. If the circle is at rest at time $t = 0$, the magnetic field & dipole moment at the centre are respectively

$a)\ \frac{\mu_o}{4\pi}\left(\frac{at^2+2bt}{2}\right), \frac{at^2+2bt}{4}$ $b)\ \frac{\mu_o}{4\pi}\left(\frac{at^2+2bt}{4}\right), \frac{at^2+2bt}{2}$ $c)\ \frac{\mu_o}{4\pi}\left(\frac{at^2+2bt}{2}\right), \frac{at^2+2bt}{2}$
$d)\ \frac{\mu_o}{4\pi}\left(\frac{at^2+2bt}{4}\right), \frac{at^2+2bt}{4}$

Q6. A unit electric charge is uniformly distributed over the curved surface of a circular cylinder of unit radius & finite length. Cylindrical surface is open at both the ends & is rotating with a time dependent angular speed of $pt^3 - qt^2 (\forall\, p > q > 0)$ about its longitudinal axis. The magnetic field & dipole moment at $t = 2$ on the axis are respectively

$a)\ \frac{\mu_o}{4\pi}(2p - q),\ 8p - 4q$ $\quad b)\ \frac{\mu_o}{4\pi}(2p - q),\ 4p - 2q$ $\quad c)\ \frac{\mu_o}{\pi}(2p - q),\ 4p - 2q$

$d)\ \frac{\mu_o}{\pi}(2p - q),\ 8p - 4q$

Q7. An electric charge of 2 units is uniformly distributed over the surface of a unit sphere rotating with a time dependent angular acceleration of $3t^3 - t + 2$ about its diametral axis. The rate of change of magnetic field & dipole moment at $t = 1$ on the axis are respectively

$a)\ 4\mu_o, \frac{4}{3}$ $\quad b)\ 3\mu_o, \frac{4}{3}$ $\quad c)\ 2\mu_o, \frac{8}{3}$ $\quad d)\ \mu_o, \frac{8}{3}$ $\quad e)\ none$

Q8. A unit electric charge is uniformly distributed throughout a solid cylinder of unit radius & finite length, initially at rest, rotating with a time dependent angular acceleration of $6t^2 - 2t$ about its longitudinal axis. The magnetic field on the axis is

$a)\ \frac{\mu_o}{2\pi}(2t^3 - t^2)$ $\quad b)\ \frac{\mu_o}{4\pi}(2t^3 - t^2)$ $\quad c)\ \frac{\mu_o}{2\pi}(6t - 1)$ $\quad d)\ \frac{\mu_o}{4\pi}(6t - 1)$ $\quad e)\ none$

Answers

Subjective Questions

Q1. $B = 40\,\mu_o,\ \ M = 80\pi$

Q2. $B = 8.6 \times 10^{-6}\,T$ along +ve z-axis, $M = 43\ A/m^2$ along +ve z-axis

Q3. $B = \mu_o(t^3 + t^2)$, $M = 54\pi(t^3 + t^2)$

Q4. $B = 8.98262 \times 10^{-7}\,T$ along +ve z-axis, $M = 168\ A/m^2$ along +ve z-axis

Q5. $\vec{B} = \frac{\mu_o}{\pi}(4t^2 - 3t)\hat{k}\ T$

Q6. $\vec{B} = -5.8273 \times 10^{-5}\hat{k}\ T$

Q7. $\vec{B} = 3.8062\hat{k}\ mT$, $\vec{M} = 2026.5124\hat{k}\ Am^2$

Q8. $\frac{d\vec{B}}{dt} = 5 \times 10^{-6}\hat{k}\ T/sec$, $\frac{d\vec{M}}{dt} = 5550\hat{k}\ Am^2/sec$

Multiple Choice Questions

Q1. $b)$ Q2. $c)$ Q3. $a)$ Q4. $d)$ Q5. $a)$ Q6. $c)$ Q7. $d)$ Q8. $a)$

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Email:hcrajpoot.iitb@gmail.com

Author's Home Page: https://notionpress.com/author/HarishChandraRajpoot

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